A Convex Quadratic Programming Approach for Link Shape Optimization with Dynamic Considerations

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In this research it is shown that optimizing the shape of mechanism links so as to reduce the forces and moments in the mechanism can be formulated as a convex quadratic program.

Dynamic balancing is the art of reducing (or eliminating) the *dynamic reactions* involved with the rotary-to-oscillating motion conversion, imposed by a mechanism. The dynamic reactions considered here are (i) the *shaking force* and (ii) *shaking moment* transmitted to the supporting frame, as well as (iii) the input or driving torque, required to drive the mechanism with constant drive speed. The motivation for dynamic balancing is twofold. First, shaking moments and shaking forces cause vibrations of the supporting frame (which in turn cause noise, wear, fatigue, ...), and should therefore be minimized. Second, highly peaked driving torques necessitate large flywheels (compromising the machine's start/stop behavior) or high actuator torques (and hence a large motor) in order to obtain a constant drive speed.

Spring addition and counterweight addition constitute two prominent dynamic balancing approaches. The presented research focuses on counterweight addition for dynamically balancing planar mechanisms. Determining the counterweight parameters constitutes a nonlinear optimization problem, for which many approaches exist, see e.g. the comprehensive surveys [1, 2].

In the case of a planar mechanism, nearly all these approaches directly optimize, per counterweight, a set of four parameters that completely characterize the mass properties of the counterweight. This results in an optimization problem with a rather small amount of optimization variables, but nonlinear (nonconvex) structure. Besides the problem of local optima, another drawback of this approach is the difficulty to impose constraints related to the actual shape of counterweight. For instance, restrictions on the shape and location of the counterweights in order to avoid collisions of the moving links.

An alternative approach is proposed in [3], in which a grid of mass elements, attached to each of the moving links is optimized. The resulting optimization problem is, again, nonlinear (nonconvex). The main contribution of the presented research is the proof that this problem can be reformulated as a convex optimization problem, more specifically a large-scale convex quadratic program. Convex optimization problems are nonlinear optimization problems that have the unique property that all local optima are also globally optimal. As a result, it is guaranteed that the global optimum is found, with great speed and without requiring an initial guess. These most interesting features are numerically illustrated by applying the developed methodology to the problem of dynamically balancing a planar crank-rocker four-bar mechanism.

References

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