Reachability and Control Synthesis for Piecewise-Affine Hybrid Systems on Polytopes

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Abstract

In this talk, we present a method for the synthesis of control laws for piecewise-affine hybrid systems defined on polytopes. This type of hybrid systems consists of a discrete automaton, with a continuous-time affine system on a (full-dimensional) polytope at each discrete mode, and a switching mechanism between the discrete and continuous dynamics. Control objectives for this class of systems include stability, safety, performance optimization, and robustness. In many of these problems, the notion of reachability plays a central role: does there exist an input trajectory that guarantees that the system transits from a given initial state to a required terminal state? Unfortunately, in full generality, this reachability question is undecidable.

In order to obtain applicable results, we choose a rather pragmatic approach. A set of sufficient conditions for reachability is presented, and at the same time a procedure is developed to synthesize feedback laws for the problems of reachability and stability, while guaranteeing safety. Obviously, the proposed method is conservative because it is based on sufficient conditions for reachability. Its main advantage is the fact that the approach is constructive: the sufficient conditions can be checked algorithmically, and if they are met, a constructive algorithm can be applied to obtain a control law that realizes the control objective.

The approach to control synthesis for hybrid systems presented in this talk is based on an idea of decomposition. The overall problem is decomposed into two reachability problems, one at the continuous and one at the discrete level.

At the continuous level the reachability problem can be stated as a *control-to-facet* problem: determine an affine feedback such that every trajectory of the closed-loop system reaches a particular exit facet, or one of the facets of a set of exit facets, in finite time. Conditions for the solvability of this problem can be stated in terms of inequalities on the elements of the matrix and vector that describe the affine feedback. The blocking of non-admissible exit facets yields a set of linear inequalities, and in order to guarantee that an exit facet is reached in finite time, a set of bilinear inequalities has to be satisfied. If the state polytope is a full-dimensional simplex, both conditions can be stated in terms of linear inequalities on the inputs at the vertices of the simplex. This reformulation is based on the fact that an affine function on a full-dimensional simplex is uniquely determined by its values at the vertices. The main advantage is, that this set of linear inequalities can be solved using existing algorithms for polyhedral sets.

At the discrete level, the reachability problem is to determine a path from the initial discrete state to the terminal discrete state without visiting unsafe states in a (non-deterministic) finite state automaton. This problem is solved using a backward recursion algorithm, similar to the ones used in dynamic programming.

In the talk, the main emphasis will be on the solution of the control-to-facet problem in continuous time, and on the question how in each discrete location an affine feedback should be chosen, in order to guarantee that the overall closed-loop hybrid systems satisfies the a priori given control objectives.

References

[1] L.C.G.J.M. Habets and J.H. van Schuppen. A control problem for affine dynamical systems on a full-dimensional polytope. *Automatica*, 40:21–35, 2004.

[2] L.C.G.J.M. Habets and J.H. van Schuppen. Control to facet problems for affine systems on simplices and polytopes - With application to control of hybrid systems. *Proc. 44th IEEE Conference on Decision and Control*, pp. 4175–4180. IEEE Press, New York, 2005.

[3] L.C.G.J.M. Habets, P.J. Collins and J.H. van Schuppen. Reachability and control synthesis for piecewise-affine hybrid systems on simplices. Accepted for publication in: *IEEE Transactions on Automatic Control*.