Constrained state estimation: a critical evaluation

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Abstract

Kalman filtering for state estimation has been studied intensively in the past several decades. However, a drawback of the technique is its inability to take hard constraints on the state or disturbance variables into account, while they often cannot be neglected in practical situations. Many alternative approaches are available for constrained state estimation. The state constraints can be incorporated in the Kalman filter by projecting the unconstrained Kalman filter estimate onto the state constraint surface in each time step. However, this is at most an ad hoc solution to a difficult problem. A class of estimation methods that seem to offer a better solution is moving horizon estimation. A critical assessment will be given on the advantages and disadvantages of both approaches.

1 Kalman filtering with inequality constraints

The Kalman filter is the standard choice for estimating the state of a linear system when the measurements are noisy and the process disturbances are unmeasured. One reason is that it possesses important theoretical properties such as stability and optimality. A second advantage is its low computational complexity. Often additional insight about the process is available in the form of inequality constraints. The constraints can be incorporated in the Kalman filter by projecting the unconstrained estimates onto the constraint surface in each time step. However, it can be shown that the Kalman filter is equivalent to a moving horizon estimator with a horizon of one. Consequently the information which is comprised in the inequality constraints is only taken into account for one time step.

2 Moving horizon estimation

Moving horizon estimation (MHE) methods can be motivated as the dual formulation of model predictive control (MPC) for state estimation. The strategy of MHE is represented in Figure 1. In each time step a fixed-size optimization problem is solved. Past data is summarized by an *arrival cost*. Constraints are taken into account over a trajectory of states. It is shown in this presentation that this approach leads to superior estimation performance.

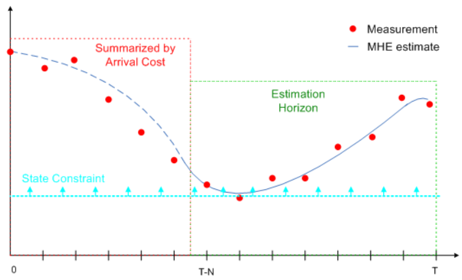

Figure 1: Representation of the strategy for moving horizon estimation.

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